

## MATHEMATICAL MODEL OF THE OBJECT OF THE CRYOGENIC PHYSIOTHERAPY ACTION

The assumptions, made during the construction of the physical layer model of the patient body, make it possible to proceed to the analytical description of the nonstationary transfer of heat through the three-layered structure, which contains the internal sources of heat. In this setting the process in question has many similar signs with the tasks, extended in the technology of low temperatures, according to the calculation of temperature fields, that are formed in the moistened materials under the action of external cooling. The noted similarity makes it possible to consider the known difficulties of the mathematical simulation of the processes of freezing.

For investigating the processes, which take place in the integumentary cloths in the course of cryo-therapeutic action, is used the mathematical model, based on the solution of the differential equation of energy with numerical methods. In general form the equation of energy is written as follows:

$$\rho \frac{\partial h}{\partial \tau} = - \left( \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) + q_v, \quad (2.2.1.)$$

The basic transfer of heat proceeding along the axis  $X$ , which is perpendicularly to the surface of object, whose length on two orders is more than thickness of the layer, in that case the process is considered as one-dimensional:

$$\rho \frac{\partial h}{\partial \tau} = - \frac{\partial q_x}{\partial x} + q_v \quad (2.2.2.)$$

where  $h$  - enthalpy (heat content) of the material, which forms layer;  
 $q_x$  - heat flux along the normal plane of the layer of coordinate;  
 $q_v$  - quantity of heat, released by internal sources.

The problems, which describe the transfer of heat through the multilayers structure, which contains the internal sources of heat, usually solve with the use of numerical transformation diagrams of the differential equation of thermal conductivity. For constructing the mathematical model of the object of cryogenic action taking into account the moisture content of layers and high probability of phase transitions to it is more preferable select the numerical solutions of the one-dimensional equation of energy. In this case increase in the heat content at the elementary points of the grid of partition is calculated. The changed values of temperatures of elementary points are calculated from the new values of enthalpies. This approach excludes the disturbances of elementary heat balance.

During the replacement in the equation of energy of derivatives it is obtained by difference approximations:

$$(h_i' - h_i) \rho \Delta x = (q_{i+1} + q_{i-1} + q_v \Delta x) \Delta \tau \quad (2.2.3.)$$

where -  $h_i'$ ,  $h_i$  respectively flowing and following on the time of the value of enthalpy at the  $i$ -point;

$q_{i+1}$ ,  $q_{i-1}$  - respectively the supply of heat from the previous and subsequent nodal points;

$q_v$  - supply of heat from the internal sources in the volume, in reference to the  $i$ -point.

$$q_v = \rho \cdot q_g \quad (2.2.4.)$$

where  $q_g$  - specific heat emission of the cloth (see Tab.2.1.1)

The supply of heat along the axis  $X$  is determined by Fourier law:

$$q_{i+1} = \frac{\lambda \cdot (T_{i+1} - T_i) \cdot F}{\Delta x} \quad (2.2.5.)$$

$$q_{i-1} = \frac{\lambda \cdot (T_{i-1} - T_i) \cdot F}{\Delta x} \quad (2.2.6.)$$

where  $\Delta x$  - distance between the points (step of partition);  $F$  - area of elementary section, through which is transferred the heat (for the one-dimensional model  $F = 1$ ).

From the external surface of the object being simulated (with  $i = 1$ ) the heat is removed by means of the convective heat exchange:

$$q_{i-1} = \alpha \cdot F \cdot (T_1 - T_i) \quad (2.2.7.)$$

where  $\alpha$  - heat-transfer coefficient;  $T_1$  - temperature of the heat-transferring medium.

Expression (2.2.7) is the boundary condition, which determines interaction of the object with the gaseous coolant being simulated.

Boundary conditions from the side of the nucleus of the patient body are determined by the internal condition of safety of cryogenic physical therapy action, and also by the thermal task of procedure. Since action is directed toward the local, surface cooling of the epithelial layer, which contains cold receptors; therefore the depth of the propagation of the zone of supercooling cloths is limited to the thickness of layer. It is a priori assumed that the nucleus of organism preserves isothermicity, therefore, at significant removal from the surface, the temperature of body is stable:  $t_b = \text{const} = 37,0^\circ\text{C}$ .

This assumption makes it possible to determine the condition of uniqueness for the internal boundary of the layer of the biological tissues being simulated:  $t_n = \text{const} = 37,0 \text{ } ^\circ\text{C}$ .

The number of elementary sections, situated in the layer with the step of the partition  $\Delta x = 0,5 \cdot 10^{-3} \text{ m}$  comprises not more than 80. Then the total number of elementary sections is received as  $n = 100$ .

Initial conditions for the mathematical model of the layer of integumentary cloths are determined by physical model. In accordance with the adopted assumptions for the fulfillment of conditions of uniqueness it suffices to assign the temperature distribution in three cover forming cloths with the standard conditions.

The heat balance of elementary section makes it possible to calculate the values of the enthalpies of nodal points at the following temporary level. We obtain from equation (2.2.3):

$$h_i' = h_i + \frac{(q_{i+1} + q_{i-1} + q_v \Delta x) \Delta \tau}{\Delta x \rho} \quad (2.2.8.)$$

The new values of the temperature are calculated by the values of enthalpies  $T_i' = f(h_i')$ .

In accordance with the physical model it is necessary to determine the sizes of cover forming layers. Since the thickness of the layer of the epithelium and adipose tissue is important subjective sign, in the mathematical model the possibility of the variation of these parameters in the limits, determined by physical model, is provided.

To account for the structure integumentary of the layer being simulated it is necessary prior to the beginning of calculations to determine the selected thicknesses:  $l_e$  – the thickness of the epithelium,  $l_f$  – the thickness of adipose tissue.

From the assigned thicknesses the mathematical model of integumentary cloths calculates the coordinates of the boundaries of the layers:  $x_1 = l_e$  and  $x_2 = x_1 + l_f$ , and then forms the integral massif ns, which contains the index of – the sign of the layer being simulated:

- if  $1 \leq x_i < x_1$ , then  $ns = 2$  (epithelium),
- if  $x_1 \leq x_i < x_2$ , then  $ns = 3$  (adipose tissue),
- if  $x_2 \leq x_i$ , then  $ns = 1$  (muscle).

The numbers of the nodal points, which correspond to the boundaries of the layers, simultaneously are calculated and are memorized:

$$n_1 = \frac{x_1}{\Delta x}, \quad n_2 = \frac{x_2}{\Delta x},$$

The massif of the initial values of temperatures is formed taking into account the assigned geometric structure of the unit being simulated. For these purposes is used the information about the temperature of the cloths, given in chapter 2.1. Changing the value variable  $i$  in the range from 1 to  $n$ , it is possible to calculate the values of the temperature at the moment of the time of  $\tau=0$

$$\text{if } ns = 2, \text{ then } T_i = T_3,$$

$$\text{if } ns = 3 \text{ then } T_i = T_3 + \frac{(T_M - T_3) \cdot (i - n_1 - 1)}{(n_2 - n_1 - 1)}$$

$$\text{if } ns = 1 \text{ then } T_i = T_M.$$

Initial heat content for each nodal point also considers the property of the layer, in which this point is located, for example, for the fatty layer:

$$ns = 2 \Rightarrow h_i^* = q_2^* + h_{2-0} + (T_i^* - T^*) \cdot c_2,$$

where  $q_2^*$  – the heat of the defrosting of adipose tissue,

$h_{2-0}$  – the enthalpy of adipose tissue at  $T=263K$ ,

$T^*$  – the temperature of defrosting.

For all further calculations of the value of variable  $ns$  they make it possible to use in the design characteristics, which correspond to this type of cloths.

The described algorithm of shaping of the initial massifs of temperature and enthalpy of layers automatically is carried out at the beginning of numerical experiment.

The observance of the conditions of hypothermal safety is the most important requirement, presented to the technology of cryo-therapeutic action. In the mathematical model the calculation of these conditions is ensured with the aid of the computable number of the point, which corresponds to the boundary of fatty and muscular layers. Checking the external and internal conditions of the hypothermal safety is provided at each new temporary step:

$$T_{i=1} > 271K \text{ и } T_{i=n_2} > T_M - 1.$$

The disturbance of these conditions causes the curtailment of numerical experiment.